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Quadratic conservation laws and collineations: A discussion

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Abstract

Every second order system of autonomous differential equations can be described by an autonomous holonomic dynamical system with a Lagrangian part, an effective potential and a set of generalized forces. The kinematic part of the Lagrangian defines the kinetic metric which subsequently defines a Riemannian geometry in the configuration space. We consider the generic function $I=K_{ab}(t,qc)\dot{q}^a\dot{q}^b+K_a(t,qc)\dot{q}^a+K(t,qc)$ and require the quadratic first integral condition $dI/dt=0$ without involving any type of symmetry Lie or Noether. Condition $dI/dt=0$ leads to a system of equations involving the coefficients $K_{ab}(t,qc), K_a(t,qc), K(t,qc)$ whose solution will produce all possible quadratic first integrals of the original system of autonomous differential equations. We show that the new system of equations relates the quadratic first integrals of the holonomic system with the geometric collineations of the kinetic metric and in particular with the Killing tensors of order two. We consider briefly various results concerning the Killing tensors of second-order and prove a general formula which gives in the case of a flat kinetic metric the generic Killing tensor in terms of the vectors of the special projective algebra of the kinetic metric. This establishes the connection between the geometry defined by the kinetic metric and the quadratic first integrals of the original system of differential equations. © 2018 Elsevier B.V.

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